A few remarks on near field electromagnetic energy transmission

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Abstract

The interest for energy transmission toward mobile devices boosts the studies about near field non-radiant energy transmission. Some configurations in patents or papers refer to a longitudinal transmission mode. Scalar wave incompatible with Maxwell's equation are suggested.
The authors, after the proposal of a new taxonomy, close to the one used in continuous media, show that devices and experiments could all be interpreted within the classical mathematical frame.
Nevertheless the macroscopic near field seems to possess a mechanical dimension irreducible to the standard interpretation and to require an extension of the interaction concept according to the Mach's principle. Furthermore the local energy flux concept becomes ambiguous.

1 – Preliminaries

Electromagnetic fields enable since many years communications with million kilometres distant exploration vessels, they are now more and more used in near fields applications.
Simultaneously with a signal transmission realised with extremely low efficiencies, it is often question of energy transfer in order to obtain autonomous distant apparatus even when an energy storage unit is not used [1]. Recently a new apparatus was studied in the MIT in order to transfer energy at intermediate distances with a good efficiency in a non radiant manner [2].
To do so this apparatus and many others use simple coils or flat printed spiral type antennas. The two coils are often arranged in a coaxial manner. These systems, in the frequency range used, behave more like air transformers than like emitting-receiving systems.
The confusion is often made between near field and electromagnetic waves regimes. However it is possible to find as much differences between these two behaviours as between an air flow and an acoustic wave.

An expression like Radio Frequency Identification (RFID) could be often better replaced by: "weakly radiant inductively coupled identification systems". This confusion suggests that waves are exchanged between the elements and can be attributed to the wave focused evolution of the teaching of electromagnetic phenomena.
This orientation toward an increasing presence of the wave interpretation, leaving on the side near field aspects, has started with the electromagnetic unification and was reinforced during the elaboration of the standard model by Richard Feynman conviction that all observed electric and magnetic phenomena can be reduced to photons exchanges.
Aside from any polemic, it is useful to remind prosaically that unification doesn't mean identity. Electro-weak unification that describes in the same formalisms both electromagnetic and weak interaction has not changed our way to see electromagnetism. In the same manner the electromagnetic unification should not hide the existence or purely magnetic or electric behaviours. In a unified representation, the existence of EM waves should be introduced only as a special configuration and not the general rule.
The distinction between near and far fields is also confusing, if the concept has a clear mathematical meaning, it cannot be used alone to characterise a given apparatus' behaviour. Some systems radiate energy far away and others do not or very little, for both of them the near and far field can be defined. Non-radiant devices are usually described by dynamic theoretical models improperly called quasi-static approximations. The EM waves, solutions of some equations derived from the Maxwell ones, can also be considered as approximate solutions of the whole problem. The formal discontinuity between electric, magnetic and EM regimes are known but often discarded in usual high school teaching curses [4]. In order to get read of the great confusion existing in the field and to consider safely in one hand practical situations such as biological effect of quasi-static fields and EM radiations and in other hand theoretical concerns such as longitudinal waves existence or standard model completeness, it seems first necessary to think on an appropriate vocabulary associated to a clear taxonomy.
Compact expressions such as "electrostatic motor" are worrying. Reversely if we consider a situation similar to the study of an air flow around a fan, we are presently forced if we wish to be precise enough, to use complex expressions such as "near field study of a quasi-stationary type apparatus".

2 – A clear classification in the conceptual domain.

The classification that we are about to suggest is similar to the one existing in fluid mechanics. Maxwell equations, originally result from the gathering of empirical results from various experiments, they are known to have a mathematical structure close to the one involved in the continuous material media description. This leads to various similarities between these two domains [5].

Like fluid mechanics, EM equations lead to two classes of solutions with very different properties. These solutions are on one hand EM waves and on the other hand local quasi-static fields. The first class arise in situations where one considers phenomena that are propagating far away and independently to the ulterior behaviour of the source and then accordingly to the particles in the standard model. The associated domain is the equivalent for EM fields to acoustics toward fluid mechanics. The second class of solution arises in the proximity of sources and is attached to the mathematical scalar potentials concept. This class is similar to the study of material flows, its present a mechanical aspect and its study implies global consideration involved by non-linear effects[1].

In the standard model paradigm, phenomena belonging to the second class are regrouped under the general term of 'interaction'[2]. In quantum descriptions, photons are the EM interaction's vectors. In the standard model the usual capacitor would be a photon box like system. However the photons couldn't be like particles only inside the box, they rather have to stay mainly outside in order to explain the resulting attracting force as photons impacts on electrodes. It is not proved that Quantum electrodynamics that deals with extremely localized binary interaction in space and time can be used to describe this type of situations. Moreover this theory uses to describe the near field of the electron the classical Coulomb's potential; it is likely very strange that it succeed to describe its own basis[3].

This difficulty of quantum theory was very soon evoked by Rosenfield in the conclusion of his 1932 paper on the quantum theory formalism he contributed to develop for a large part[6]: "But nothing allow us to think that the same method is able to give an adequate description of material particles interactions, or to explain the same idea in other terms, that the photon concept (which is resulting immediately from the quantification of the electromagnetic field) could be used to analyze other fields than the purely radiant ones".

Electric and Magnetic near fields cannot be clearly described as resulting from an infinity of multiple punctual EM interaction between virtual particles but are more likely the effects at large scale of a fundamental electro-mechanic interaction (between EM field and matter) which cannot be described in any present quantum form and do not even fit in the standard model frame.

To illustrate simply this fundamental difficulty lets consider two electrons initially at rest alone in the vacuum. The best model to describe these two electrons is the Coulomb's force and associated field. It is worth saying here that such near fields problems need, to be closed, the adjunction of the Lorentz's force [7] associated to the mechanical laws of movement or an equivalent principle such as a friction between charges and neutral matter. As a result near field problems are always either electro-mechanical or magneto-mechanical. Let's come back to our two electrons and wonder in the possibility to define any proximity or contact criterion. We already know that the solid contact has not a real meaning at the atom scale; the electrons seem to have no physical extension. The Coulomb's field do not present any transition in order to allow a near field definition. As a result our two electrons are in coulombian interaction whatever the distance between them. If the solid contact is defined by the existence of an electrostatic repulsion between the electrons of the two objects, then our two electrons are in solid contact whatever the distance between them!

Suppose now that one of the two electrons is oscillating, we can now associate to this electron a length: the wavelength in vacuum, and now define a proximity criterion for this small movement which is superposed to the original position of the electron. The second electron will be said to be in the near field of the first one, if the distance between them is shorter than the wavelength. If the distance is greater we will say that the two electrons

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[1] In EM non-linearity arise when the Lorentz's force is introduced because of its quadratic form. Such quadratic non-linearity arises in field/matter interactions. At the macroscopic level they appear in electro mechanic or magneto mechanic devices and more generally in the so-called near field domains.

[2] However there is here a fundamental difficulty that seems to be under evaluated up to now. The interactions occurring in the macroscopic near field involve continuously a great number of particles and are still present on distance sometime above one meter. We are very far here from the frame of two particles interacting punctually in time at the nanometric level and then behave in an autonomous and peaceful way the rest of the time.

[3] This point cannot be discussed in the Coulomb's gauge formulation of Quantum electrodynamics. In the Lorentz's gauge formulation however it is possible to see the Coulomb's potential as a macroscopic result from contributions of an infinite number of virtual particles. These particles even if they are handsome for computation of some electron's momentums, do not bring any new perspective on the fundamental nature of EM fields.

are interacting though electromagnetic waves. We see that the near field range is a relative notion, for low frequencies the range of the near field interaction could be extremely large. Mach was referring to such a mechanism when in a notorious principle he suggested that our near space is structured by the far objects of the universe.
In practise to avoid incoherent arguments we can only rely on the content of Maxwell equations and we should avoid to evoke any reference on propagation of anything between the different parts of an apparatus involving such near field interactions. In fact all this is nothing more than the equivalent obvious fact that a local air flow cannot be reduce to a summation of acoustic waves!

3 – Taxonomy and terminology

If the near field general behaviour remains quite complex, it is widely known that when the system remains in what is usually defined as quasi-static regimes, natural and simple solutions that do not involve complex electromagnetic-mechanical equations are available[1].
Such regimes are characterized by the fact that one of the two coupling terms in the Maxwell equations is null or can be neglected. This simplification leads to reduced equations that do not allow propagation of waves. To explain it in another way, such situations are obtained when it is possible to consider that the interactions between every point of the involved domain seen as a non-dissociable assembly are instantaneous[2]. The quasi-static electric limit enables the mathematical description of capacitors and more generally the study of dipoles or multi-poles coupled in near field whereas the quasi-static magnetic limit enables the description of coupled magnetic dipoles or multi-poles (rotating machines, transformers…). In both cases the energy whether it is in the electric or the magnetic form stays stored in the vicinity of the sources and do not propagates at distance. In the fluid domain these situations correspond to the hydrostatic or more generally the one of local flows. Thus it is possible to find a disturbing analogy between an electric charge and a hurricane.

We propose to call respectively "influence" and "induction" or simply electric and magnetic regimes the two non-radiant limits and to keep "EM waves" for phenomena related to the solutions of a wave equation. Then at a short distance to a large source of electric field, it will be sometime possible to describe the situation in terms of electric regime or in terms of waves if the size of the system is large compared to wavelength. At larges distances waves will only remain in all situations[3].
Apparatus relative size compared to wavelengths is a central technical characteristic linked to it main behaviour. If its relative size is small the device will mainly store potential energy, whereas in the reverse case it will mainly radiate the energy as waves. We propose to call the first class of apparatus: "Influence or induction devices" and the second one: "radiant devices".
In accordance with usual electrostatics conventions it is possible to say that the usual capacitor is a total influence device whereas interacting dipoles are only partial influence ones.
In the induction frame, if one remembers that the total influence case corresponds to a situation where two opposite charges exchanged all their fields' lines then the magnetic equivalent correspond only to the case of a magnetic dipole alone.
In both cases in order to transfer energy at distance one needs to associate a distant load dipole to the generator one. If such coupled structures associated to a large number of devices are well known in the magnetic regimes it is very unusual to see equivalent electric structures.

Although extracted from the general frame of Maxwell-Lorentz's equations, Influence and Induction can present very disturbing aspects when seen with a mind inspired with the wave paradigm.
The conceptual differences between waves' behaviour and the Induction and Influence domains are clearly seen when one considers longitudinally coupled devices. In such cases, because an oscillating dipole radiates its energy transversally and the Poynting vector which represents the energy density flux is null along the dipoles axis, there are in the wave paradigm no waves travelling between the generator and the load. On the opposite if one considers the distant mechanical work of the electric force the difficulty vanishes[4].
With Influence and Induction, energy is longitudinally transported between to near dipoles in the same manner that it is in an electrical wire. This is easy to understand if one remembers that matter is mainly constituted of

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[1] i.e. without to solve the entire Maxwell-Lorentz's equations.
[2] These equations can also been obtained from Maxwell equations in the limiting cases of infinite light speed.
[3] Unlike for gravitational effects universe is electrically neutral at large scales.
[4] Conceptually it is not the fact that the energy density flux is not defined locally which poses problem but more generally that the movement of energy cannot be clearly associated with a given flux of photons. To resume, if one accepts to come back to the former representation in terms of force without trying to interpret this one in the standard model frame the difficulty vanishes but the mystery remains.

vacuum and that inside matter the electrons act mechanically one between another at distance through the coulomb's force.
Then even if it is still possible to distinguish a generator device and a load one, it is not possible to reduce the situation to some energy travelling independently from source and destination. The emission and reception concepts became ambiguous (see appendix1). One has to represent the system as a whole with a global coupling. It cannot be anymore structured in separate and somehow independent units. The words: emitter, receiver, emission, reception should be totally avoided. If it is still possible to use the word transmission in a mechanical meaning, it seams better to use the word transportation. Consequently, the vacuum appears like a medium that allows wireless and non-radiant energy transportation.

4 – Various experiments, articles and patents critical analysis

After the previous remarks, we can analyse polemical subjects to shows that they are in fine all belonging to the same Maxwell's equation frame.

4.1- Tesla

The following apparatus was realised by Tesla during his Colorado Springs period (1899).

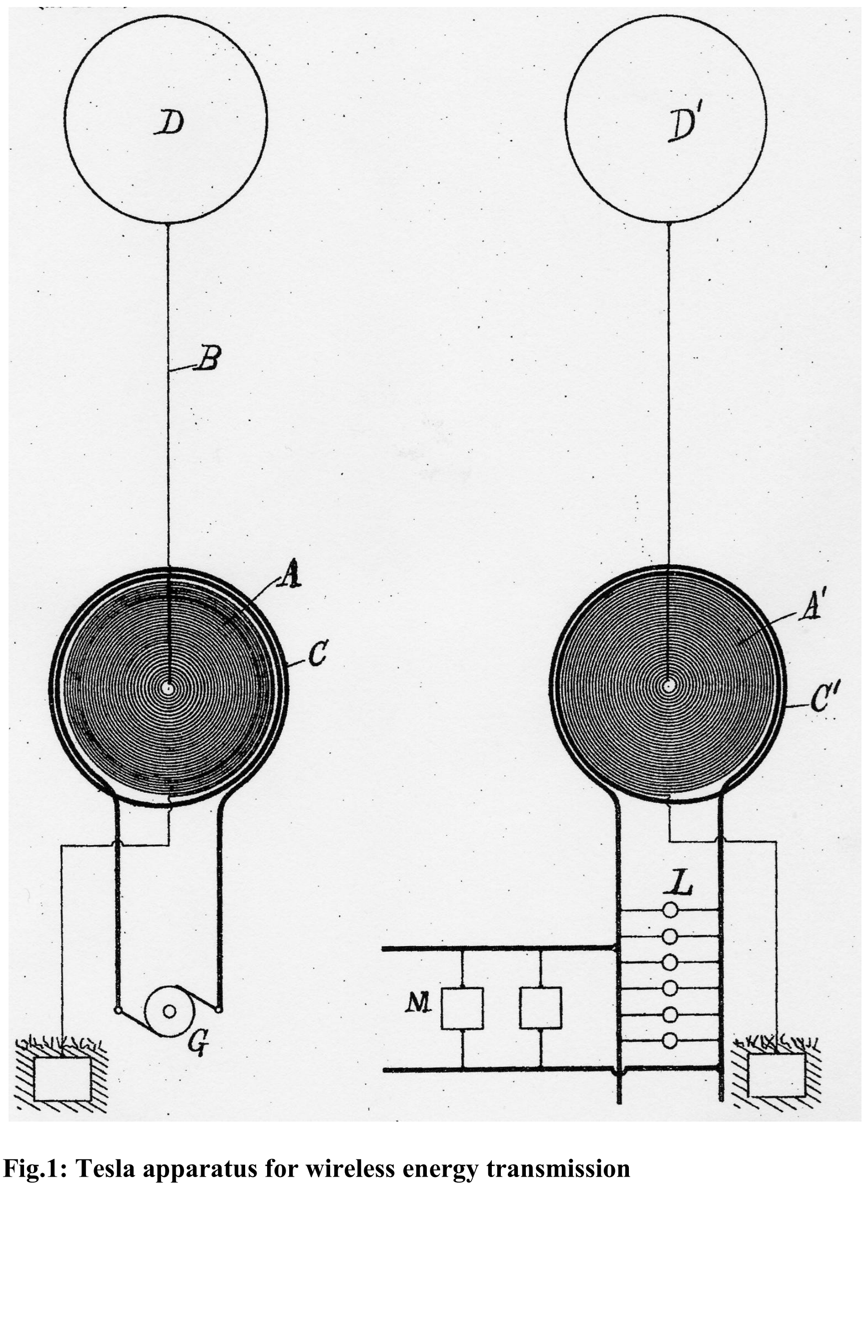

Fig.1: Tesla apparatus for wireless energy transmission

Although Tesla's objective was wireless energy transmission and the preceding apparatus was patented with a clear title [8], he deposited a second patent soon later for a device very similar to the previous one but claiming this time for EM waves. Tesla has probably heard of Marconi's work meanwhile.
The generator G is made of a 40nF capacitor charged with a 5kV alternated current generator and discharged at 5 kHz in the coil C trough a mechanical rotating switch. The secondary circuit is made of the coil A and the spherical electrode D. The primary circuit, the secondary one and the receiving one D',A' are all tuned to the same oscillating frequency of about 240 kHz.
Although the preceding apparatus interpretation is clear, it is worth noting that the wavelength of such a device is in the kilometre range and then the dipole size was several order of magnitude below the wavelength, leading to a very inefficient radiant device. Energy transmission very small in the kilometre range was more important in the decametre range. Tesla, although unaware of it, was indeed in the influence domain and then could really transfer energy with a good efficiency. In his experiments the sphere radius was about 5m and the primary power around 2,5kW then a few hundred watts could have been sent to distances around about fifty meters.

4.2-Avramenko

Avramenko has claimed through a patent [9] the discovery he though he has made of longitudinal density waves. In fact his apparatus works in the electrical influence regime and he uses the intrinsic capacity of a conductor alone without referring to it. He his wrongly thinking that the so-called waves will propagates arbitrarily far along a wire providing then a simple way to transfer energy at distance.

4.3-Meyl, Monstein & Wesley, Tzontchev et al

Relaying on Avramenko patent, Meyl [10] has though it was there a crack in the standard science and has started its own theory that was criticized by Waser, Waser himself as proposed an extension of Maxwell equations in order to include longitudinal waves behaviour [11].
Monstein & Wesley [12], criticized by Bray & Britton [13], Tzontchev et al's work [14] interpreted in an abstract manner by Onoochin [15], are all belonging to the same controversy.
If the earth is considered as a conductor, all the generator and load devices described are dipole-like structures often very similar to the Tesla apparatus. If we take into account the frequencies used, the described apparatus are mainly radiating ones. Some longitudinal effects could be observed in near field but according to the globally transverse structure they behave like classical radiant sources at large distances.
For Monstein & Wesley, apart for the omnipresent field/wave confusion, the mathematical description is evoked without rigor. For instance it is not a retardated potential that generate a wave equation like they seem to infer but a wave equation that has a solution in terms of retardated potentials. Furthermore to obtain such an equation one has to use the frame of the Lorentz's gauge and then at the same time a propagation equation is also obtained for the vector potential and the resulting wave is a classical transverse one (see appendix 2 for more explanations). The authors make then the following mistake, they keep only the scalar potential and forget to consider the vector one and then they switch back to the Coulomb's gauge which does not lead at all to such a solution when used properly.
For Tzontchev & al, the objective is to demonstrate a finite speed for the scalar coulombian potential that they thing would be in contradiction with Maxwell equations. In his analysis of this work, Onoochin demonstrates that the scalar potential is not directly tied to the longitudinal field and then Tzontchev experiment do not state a clear evidence of a failure of the original model and as a result of the existence of such scalar waves. He, by the way, comes to a related concern, a bit apart our main topic and close to Rousseaux considerations [5], the question of the physical meaning of the gauge invariance.
In the recurrent topic of the instantaneity of the near coulombian field, it is interesting to note that such a field go with (or even constitute) the electric charge in a resulting extended object and, apart from relativistic contractions results, to a perfectly rigid body[1]. The "mechanical" displacement of such a field is then bound to the constraints of general relativity and is then slower than light. To measure the propagation of such a field one should first find a way to modulate it and then in practise to be able to create or remove a charged particle in a given point of the vacuum. This is forbidden not only in the classical frame but even in all quantum ones.

[1] The origin of the quasi-instantaneous near field is strongly related to this property. In fact this field can be seen as constant and is simply accompanying or constituting the particle.

To conclude positively this paragraph, we may say that some authors have tried to catch the reader attention on some obscure fact arisen in EM phenomena when someone tries to figure the situation only in the wave paradigm. Others have tried to find a break in the wall using unreliable arguments.

5 - Could it be after all some incompleteness in the Maxwell model?

To catch some unpredicted behaviour, a spherical geometry that could not be reduced to a dipolar structure is necessary. Such a structure will 'de facto' suppress any transverse field and radiations. The simplest system consists to consider two equipotent concentric spheres submitted to varying and for instance sinusoidal potential difference. The Gauss theorem is still valid in such a dynamical situation and implies that the external field should remain null (because the internal charge is globally null). Detection of an external field that could be attached to the propagation of scalar potentials is then equivalent to demonstrate a failure of the Gauss theorem and by the way a non-null mass for the photon [16].
Such experiments were done many times in the past, the oldest one is the Cavendish Spheres [17], in a recent one the two spheres are exited at high frequencies (4 MHz) they have proved that if divergences with the Gauss theorem exist it is extremely small [18].

7 - Conclusion

Far from sources, the only solutions of Maxwell equations are waves. Near sources other solutions arise. Although they can be related to a wireless transportation of energy, these solutions cannot be described as waves nor be related to classic photons behaviour. As a result some researchers have imagined some new type of waves called 'scalar' or 'longitudinal' waves. These waves are not compatible with Maxwell equations then some researchers have sought for some incompleteness in the Maxwell system.
However, very precise experiments have shown that such a failure if existing is extremely weak and cannot be used to explain the observed near field energy transportation, then only a representation problem remains.
The present reader may found a limited interest to this approach somehow philosophical. Actual field whatever they are near or far ones verify the same set of equations and describe a same unified object with multiple behaviours. This argument of a unified structure is acceptable and we also defend it. It doesn't suppress the interest that following the mathematical formal discontinuity that is present in the Maxwell equations to clearly separate two domains with antagonistic behaviours. Such an approach allows at the same time, a simplified mathematical and conceptual approach, industrial applications through an easier understanding of the apparatus mechanism and finally a best sight on modern physics limits.
Illustrations based on fluid mechanics where used to illustrate these points. No one can deny the fact that it is possible and sometime very useful to consider a gas as a continuous medium even if at atomic scale the behaviour is described through quantification and binary interactions. This leads to, on one hand acoustic and it own properties, on the other hand to flow analysis and there own specificities. In an acoustic problem it is possible to have a local approach conform to the standard model (a wave can propagate independently to the original source it comes from) while in a flow problem the behaviour is necessary global (an obstacle in one point changes the entire flow). When near fields are considered, the same global situation arise in the EM domain whereas the underlying structure remains unclear even in quantum descriptions.
It is also possible to follow the pragmatic position by observing that actual reality is elusive and only working equations and efficient models should be preferred to obscure and complex ones. The authors think that physical science essence is a permanent feedback between a given perception of the world and its translation into mathematical forms. They think that a new paradigm is a need to a correct near field interpretation and that it modifies deeply the standard description and this goes well farther than the simple EM domain.
Following Einstein, the ether concept is not necessary to explain special relativity and was suddenly banished although it could remain a potentially fructuous one. Later on Einstein tried unsuccessfully to resuscitate it as the substratum of the global theory of general relativity.
Presently, the simple idea to reintroduce the concept of a continuous underlying substratum is hard to formulate even for a cords theory specialists. Although manipulating such entities they prefer to call them with the more abstract terms of varieties keeping the confusion between the coordinates and the underlying object under study.
The authors think that at least in a practical point of view, following the fluid mechanics analogy and whatever the underlying structure, an EM field description in terms of apparent stress of an undefined continuous medium is a conceptual tool useful to understand the near field behaviour and the wireless energy transfer its enable.

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Appendix 1:

Energy density and flux indeterminations

From Maxwell equations (1) and a classic vectors' algebra theorem (2)

$$\left.\begin{array}{ll} div\,\boldsymbol{D} = \rho & \boldsymbol{rot\,E} = -\dfrac{\partial \boldsymbol{B}}{\partial t} \\ div\,\boldsymbol{B} = 0 & \boldsymbol{rot\,H} = \boldsymbol{j} + \dfrac{\partial \boldsymbol{D}}{\partial t} \end{array}\right\} \quad (1) \qquad div(\boldsymbol{E} \times \boldsymbol{H}) = \boldsymbol{H}.\boldsymbol{rot\,E} - \boldsymbol{E}.\boldsymbol{rot\,H} \quad (2)$$

one can get : $\boldsymbol{E}.\dfrac{\partial \boldsymbol{D}}{\partial t} + \boldsymbol{H}.\dfrac{\partial \boldsymbol{B}}{\partial t} + div(\boldsymbol{E} \times \boldsymbol{H}) = -\boldsymbol{j}.\boldsymbol{E}$ (3)

Usually the following quantities are defined:

$$\frac{\partial u}{\partial t} = \boldsymbol{E}.\frac{\partial \boldsymbol{D}}{\partial t} + \boldsymbol{H}.\frac{\partial \boldsymbol{B}}{\partial t} \quad (4) \qquad \boldsymbol{P} = \boldsymbol{E} \times \boldsymbol{H} \quad (5) \qquad \frac{\partial w}{\partial t} = -\boldsymbol{j}.\boldsymbol{E} \quad (6)$$

and one gets: $$\frac{\partial u}{\partial t} + div\boldsymbol{P} = \frac{\partial w}{\partial t} \quad \text{(3bis)}$$

Relation (3bis) is the local form of energy conservation.

u is the local energy density for the electromagnetic field.
$\boldsymbol{P}$ is called: 'Poynting vector', it represents the flux of radiant energy..
w is the energy density coming from or received by the material medium.

This description needs some more explanations.
First we may remark, following Labarthe [19], that if we consider any arbitrary vector field $\boldsymbol{T}(\mathbf{r},t)$, the following quantities: $u_1 = u - div\boldsymbol{T}$ et $\boldsymbol{P}_1 = \boldsymbol{P} + \dfrac{\partial \boldsymbol{T}}{\partial t}$, verify also the energy conservation equation(3bis). Even if we fix each of the two members of the left side of equation (3bis), any constant can be added to the energy and any rotational can be added to the energy flux.
There is no intrinsic reason to define the energy density and the energy flux as it is done in (5) and (4). There is either no ways for direct measurements of energy density and flux in near field regimes.
However the preceding choices are justified in the wave regime because the pointing vector can be associated to the propagation direction.
To give an example of the problematic quasi-static situation, let's consider the case of the constant current discharge of a capacitor placed in the vacuum.
Following the classical point of view the inter electrodes' gap is not crossed by any material current but only by a virtual one called 'displacement current' which is substituted to the material one at the frontier between the electrodes and the vacuum.
The last Maxwell's equation enables the demonstration of the total current conservation:

$$div(\boldsymbol{j} + \frac{\partial \boldsymbol{D}}{\partial t}) = 0 \quad (7)$$

Vacuum appears to be 'charged' with an energy density defined by: $u = \dfrac{1}{2}\varepsilon_0 E^2 + Cte$ (8).

For a slow discharge it is possible to demonstrate that the magnetic field is very small and from (3) we get:

$$div(\boldsymbol{E} \times \boldsymbol{H}) = -\boldsymbol{E}.\frac{\partial \boldsymbol{D}}{\partial t}$$

We also have: $div(V\dfrac{\partial \boldsymbol{D}}{\partial t}) = Vdiv\dfrac{\partial \boldsymbol{D}}{\partial t} + \dfrac{\partial \boldsymbol{D}}{\partial t}.\boldsymbol{grad}V = -\boldsymbol{E}.\dfrac{\partial \boldsymbol{D}}{\partial t}$

Then there it at least two different manners to define the energy flux leading to the same energy balance.

An even more strange aspect is that the two fluxes are at right angles. $\boldsymbol{E} \times \boldsymbol{H}$ is perpendicular to E whereas $V\frac{\partial \boldsymbol{D}}{\partial t}$ is parallel. In the classical description the energy flux is leaving the capacitor through the sides which seems at least surprising. The other description for the flux $V\frac{\partial \boldsymbol{D}}{\partial t}$ crosses the electrodes and seems more in agreement with our intuition.. The total energy flux: $V.i$ is obtain with the classic expression through an integration over the lateral cylindrical surface which same radius than the electrode where the fields are constants with the following values: $E = \frac{V}{h}$ et $H = \frac{i}{2\pi r}$.

The same result is obtained with the integration of the quantity $V\frac{\partial \boldsymbol{D}}{\partial t}$ over the upper electrode surface (the lower one is used as the null reference potential) and using the conservation of the overall current (7).

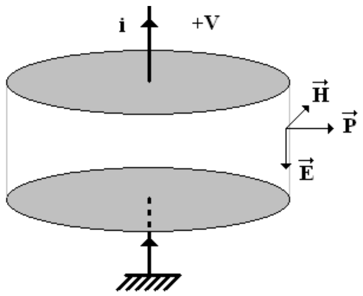

Fig. 2 Energy and flux density indetermination

Previous considerations do not concern only the above school type example they are present in other practical situations. In the study of waves in piezoelectric materials, following D. Royer & E. Dieulesaint [20], when the magnetic nergy for the waves is neglected the vector for the energy flux density is $P_j = -T_{ij}\frac{\partial u_i}{\partial t} + V\frac{\partial D_i}{\partial t}$ (9) with T_{ij} the stress tensor and u_i is the mechanical displacement vector. In such a case the direction of the P vector is different because it depends not only on the field vector E but also on the stress tensor. This result in situation difficult to interpret where energy do not propagate in the same direction than the wave vector.

The author's position confronted to this formal problem is that the concept of energy flow could be not pertinent in near field and should be replaced by a global representation.
A concluding remark is that the left terms of the energy conservation equation do concern only fields unlike the right one which concerns also the charged particles. If we consider only a particular charge its contribution to energy writes:

$$dw = \boldsymbol{j.E}d = q\boldsymbol{v.E}dt = q\boldsymbol{E.dx} \quad (10)$$

This can be interpreted as the electric force work on the particle. Then the energy conservation equation (3) indicate in a subtle manner that the interaction between to close charges in the vacuum cannot be resumed to a wave radiation problem but also implies an electromechanical aspect.

Appendix 2:

Scalar waves and Maxwell equations incompatibility

In the vacuum Maxwell equations write:

$$\varepsilon_0 \, div\, \boldsymbol{E} = \rho \quad (11) \qquad \boldsymbol{rot}\, \vec{\mathrm{E}} = -\frac{\partial \boldsymbol{B}}{\partial t} \quad (12)$$

$$div\, \boldsymbol{B} = 0 \quad (13) \qquad \frac{1}{\mu_0} \boldsymbol{rot}\, \boldsymbol{B} = \boldsymbol{j} + \varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t} \quad (14)$$

If the values of charges and currents are known, we have 8 scalar equations for 6 remaining unknown (the two field vectors) these equations are then partially redundant.
If we introduce the potentials in the following implicit manner:

$$\boldsymbol{E} = \frac{\partial \boldsymbol{A}}{\partial t} - \boldsymbol{grad}\, V \quad (15) \qquad \boldsymbol{B} = \boldsymbol{rot}\, \boldsymbol{A} \quad (16)$$

Then we obtain a more defined problem with 4 equations and 4 unknown (one scalar and one vector equation, the two other being automatically verified). A problem remain however: for one observed distribution of fields the potential are not totally defined.
If *f(r,t)* is any scalar field the following transformations leave the fields unchanged:

$\boldsymbol{A}_1 = \boldsymbol{A} + \boldsymbol{grad}\, f$ and: $V_1 = V - \frac{\partial f}{\partial t}$

Said in another way it is possible for a given situation to choose different representation in terms of potentials[1]. These changes of representations are called Gauges. To choose a Gauge is then to define an apparently artificial constraint applied to the potentials[2].
The two common gauges are the Coulomb's one: $div\, \boldsymbol{A} = 0$ (17) and the Lorentz's one:

$div\, \boldsymbol{A} + \mu_0 \varepsilon_0 \frac{\partial V}{\partial t} = 0$ (18).

The Coulomb's gauge allow a simplified description for the scalar potential: $\nabla^2 V = -\frac{\rho}{\varepsilon_0}$ (19) but give a complex form to the vector potential. It is however useful to describe the near field situations.
In the Lorentz's gauge the two potentials verify a wave equation with sources:

$$\nabla^2 V - \mu_0 \varepsilon_0 \frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\varepsilon_0} \quad (20) \qquad \nabla^2 \boldsymbol{A} - \mu_0 \varepsilon_0 \frac{\partial^2 \boldsymbol{A}}{\partial t^2} = \mu_0 \boldsymbol{j} \quad (21)$$

The potentials however remain coupled by the gauge condition (18) and this leads to transverse waves.
The Coulomb's gauge enable the study of the near electron in away in following its movement, the Lorentz's gauge enable a clear description of the waves. In this respect a gauge choice is similar to a referential choice.

To try to demonstrate the existence of scalar waves that would be compatible with Maxwell equations, Monstein & Wesley [12], take without explaining it clearly the propagation equation for the scalar potential which as we have just explained can be obtained only in the Lorentz's gauge but instead they say they use the Coulomb's gauge which do not lead to such equation. It is a sort of illusion based on a forbidden use of two simultaneous and incompatible points of view.

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[1] It is possible to also wonder if the potentials are not as real as fields which is equivalent to postulate that the system present an hidden aspect which is not accessible by the only measure of the fields defined by Maxwell equations.
[2] Rousseaux [5] suggest that the Coulomb's gauge choice could correspond to a hidden structure uncompressible regime. Such a structure is at least presently inaccessible.